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Difference cordiality of product related graphs

R. Ponraj[1], S. Sathish Narayanan[1,*] and R. Kala[2]

[1] Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India
[2] Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, India
* Corresponding Author

E-mail: ponrajmaths@gmail.com, sathishrvss@gmail.com, karthipyi91@yahoo.co.in

Abstract

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, p\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is an injective map and $|e_f(0) - e_f(1)| \le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph which admits a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordiality of torus grids $C_m \times C_n$, $K_m \times P_2$, prism, book, mobius ladder, Mongolian tent and n-cube.

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1 Introduction

Throughout this paper we have considered only simple and undirected graph. Let $G = (V, E)$ be a (p, q) graph. The number $|V|$ is called the order of G and the number $|E|$ is called the size of G. The origin of the graph labeling problem is graceful labeling which was introduced by Rosa [11] in the year 1967. Ibrahim cahit [1] introduced the concept of cordial labeling in the year 1987. M. Sundaram, R. Ponraj and S. Somasundaram defined product cordial labeling of graphs. Cordial labeling and Product cordial labeling behavior of numerous graphs were studied by several authors [2, 12, 13, 14]. On similar line, the notion of difference cordial labeling was introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [5]. In [5, 6, 7, 8, 9, 10], difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs have been investigated. Here, we investigate the difference cordial labeling behavior of prism, Mongolian tent, book, young tableau, $K_m \times P_2$, torus grids, n-cube. Also we have proved that if G_1 and G_2 are (p_1, q_1) and (p_2, q_2) graphs with $q_1 \ge p_1$ and $q_2 \ge p_2$, then $G_1 \times G_2$ is not difference cordial. Let x be any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [4].

2 Difference cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1, 2 \ldots, p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

The following results (theorem 2.2 to 2.8) are used in the subsequent section.

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Theorem 2.2. [5] Any Path is a difference cordial graph.

Theorem 2.3. [5] Any Cycle is a difference cordial graph.

Theorem 2.4. [5] If G is a (p, q) difference cordial graph, then $q \leq 2p - 1$.

Theorem 2.5. [5] $K_{2,n}$ is difference cordial iff $n \leq 4$.

Theorem 2.6. [5] $K_{3,n}$ is difference cordial iff $n \leq 4$.

Theorem 2.7. [5] The wheel W_n is difference cordial.

Theorem 2.8. [5] K_n is difference cordial iff $n \leq 4$.

The product graph $G_1 \times G_2$ is defined as follows: Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ adj } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ adj } v_1]$.

Theorem 2.9. If G_1 and G_2 are (p_1, q_1) and (p_2, q_2) graphs respectively, with $q_1 \geq p_1$ and $q_2 \geq p_2$, then $G_1 \times G_2$ is not difference cordial.

Proof. Clearly, $G_1 \times G_2$ is a $(p_1p_2, p_1q_2 + p_2q_1)$ graph. Suppose $G_1 \times G_2$ is difference cordial. Then by theorem 2.4, $2p_1p_2 - 1 \geq p_1q_2 + p_2q_1 \geq p_1p_2 + p_1p_2 \geq 2p_1p_2$. This implies $-1 \geq 0$, a contradiction. Q.E.D.

The graph $C_m \times C_n$ is called a Torus grid.

Corollary 2.10. Torus grids $C_m \times C_n$ are not difference cordial.

Prisms are graphs of the form $C_m \times P_n$. We now look into the graph prism $C_n \times P_2$.

Theorem 2.11. The prism $C_n \times P_2$ is difference cordial.

Proof. Let $V(C_n \times P_2) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(C_n \times P_2) = \{u_1u_n, v_1v_n\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\}$. Define a map $f : V(C_n \times P_2) \to \{1, 2, \ldots, p\}$ as follows:
Case 1. n is even.
Define $f(u_1) = 1$, $f(u_2) = 4$, $f(v_1) = 2$, $f(v_2) = 3$,

$$
\begin{aligned}
f(u_{2i+2}) &= 4i + 1, \quad 1 \leq i \leq \tfrac{n-2}{2} \\
f(u_{2i+1}) &= 4i + 4, \quad 1 \leq i \leq \tfrac{n-2}{2} \\
f(v_{2i+2}) &= 4i + 2, \quad 1 \leq i \leq \tfrac{n-2}{2} \\
f(v_{2i+1}) &= 4i + 3, \quad 1 \leq i \leq \tfrac{n-2}{2}.
\end{aligned}
$$

The following table 1 shows that f is a difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 2$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 1.

Case 2. n is odd.
Assign the labels to the vertices u_i and v_i $(1 \leq i \leq n-1)$ as in case 1. Define $f(u_n) = 2n$ and $f(u_{n-1}) = 2n - 1$. Clearly, this labeling is a difference cordial labeling of $C_n \times P_2$ when n is odd. Q.E.D.

Theorem 2.12. The graph $K_m \times P_2$ is difference cordial iff $m \leq 3$.

Proof. Since $K_1 \times P_2 \cong P_2$, by theorem 2.2, $K_1 \times P_2$ is difference cordial. The graph $K_2 \times P_2 \cong C_4$. By theorem 2.3, $K_2 \times P_2$ is difference cordial. $K_3 \times P_2$ is a prism and hence difference cordial by theorem 2.11. Suppose $K_m \times P_2$ is difference cordial ($m > 3$). Clearly, $K_m \times P_2$ has $2m$ vertices and m^2 edges. Using theorem 2.4, $m^2 \leq 4m - 1$. This is possible only when $m \leq 3$. Q.E.D.

Theorem 2.13. Let G be a (p, q) connected graph. If $n \geq 5$, then $G \times K_n$ is not difference cordial.

Proof. The order and size of $G \times K_n$ are np and $nq + \binom{n}{2}p$ respectively. Suppose $G \times K_n$ is difference cordial with $n \geq 5$. Then, by theorem 2.4, $nq + \binom{n}{2}p \leq 2np - 1$, $\Rightarrow 5np - 2 \geq 2nq + n^2p \geq 2n(p-1) + n^2p$. $\Rightarrow 8 \geq 10p$, a contradiction. Q.E.D.

Theorem 2.14. If G is a (p, q) connected graph. Then $G \times W_n$ $(n \geq 3)$ is difference cordial iff $p = 1$.

Proof. The order and size of $G \times W_n$ are $(n+1)\,p$ and $2np + (n+1)\,q$ respectively. Suppose $G \times W_n$ is difference cordial with $p \geq 2$. Then, by theorem 2.4, $2np + (n+1)\,q \leq 2\,(n+1)\,p - 1. \Rightarrow 2p - 1 \geq (n+1)\,q \geq 4q \geq 4\,(p-1)$. $\Rightarrow 3 \geq 2p \geq 4$, a contradiction. When $p = 1$, $G \cong K_1$. By theorem 2.7, $K_1 \times W_n \cong W_n$ is difference cordial. Q.E.D.

The book B_m is the graph $S_m \times P_2$ where S_m is the star with $m + 1$ vertices.

Theorem 2.15. The book B_m is difference cordial iff $m \leq 6$.

Proof. Let $V(B_m) = \{u, v, u_i, v_i : 1 \leq i \leq m\}$ and $E(B_m) = \{uv, uu_i, vv_i, u_iv_i : 1 \leq i \leq m\}$. For $m \leq 6$, the difference cordial labeling f is given in table 2.

n	u	v	u_1	u_2	u_3	u_4	u_5	u_6	v_1	v_2	v_3	v_4	v_5	v_6
2	4	3	1	6					2	5				
3	3	4	1	8	6				2	7	5			
4	4	3	1	5	7	9			2	6	8	10		
5	4	3	1	5	7	9	11		2	6	8	10	12	
6	4	3	1	5	7	9	11	13	2	6	8	10	12	14

TABLE 2.

Suppose $m > 6$. Let f be a difference cordial labeling of B_m.

Claim: $e_f(1) \leq m + 3$.

Case 1. Labels of u and v are consecutive numbers.

Let $f(u) = t$ and $f(v) = t + 1$. There are at most two edges uu_i and vv_j with label 1. The maximum value of $e_f(1)$ is attained when u_i and v_i receive the consecutive numbers. This forces $e_f(1) \leq m + 1 + 2 = m + 3$.

Case 2. $f(u)$ is neither successor nor predecessor of $f(v)$.

In this case, there are at most four edges uu_i and vv_j with label 1. Also, at least one of u_iv_i $(1 \leq i \leq m)$ receive the label 0. Therefore, $e_f(1) \leq 4 + m - 1 = m + 3$. Hence, $e_f(0) \geq q - (m+3) \geq 2m - 2$. This implies, $e_f(0) - e_f(1) \geq m - 5 \geq 2$, a contradiction. Q.E.D.

The graph $L_n = P_n \times P_2$ is called ladder.

Theorem 2.16. Let G be a graph obtained from a ladder L_n by subdividing each step exactly once. Then G is difference cordial.

Proof. Let $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i w_i, w_i v_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\}$. Define a map $f : V(G) \to \{1, 2 \ldots 3n\}$ by $f(u_i) = i,\ 1 \leq i \leq n,\ f(v_i) = n+1+i,\ 1 \leq i \leq n,\ f(w_i) = 2n+1+i,\ 1 \leq i \leq n-1,\ f(w_n) = n+1$. Since $e_f(0) = e_f(1) = 2n-1$, f is a difference cordial labeling of G. Q.E.D.

Next is the Mӧbius ladder. The Mӧbius ladder M_n is the graph obtained from the ladder L_n by joining the opposite end vertices of two copies of P_n.

Theorem 2.17. The Mӧbius ladder M_n is difference cordial.

Proof. Let $V(M_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(M_n) = \{u_i v_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1 v_n, v_1 u_n\}$. Clearly, M_n consists of $2n$ vertices and $3n$ edges.
Case 1. n is even.
Define a map $f : V(M_n) \to \{1, 2 \ldots 2n\}$ by

$$\begin{array}{lcll} f(u_{2i-1}) & = & 4i-3, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ f(u_{2i}) & = & 4i-2, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ f(v_{2i-1}) & = & 4i, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ f(v_{2i}) & = & 4i-1, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \end{array}$$

Case 2. n is odd.
Label the vertices u_{2i-1} $\left(1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil\right)$ and u_{2i}, v_{2i-1}, v_{2i} $\left(1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right)$ as in case 1 and define $f(v_n) = 2n$. The following table 3 proves that f is a difference cordial labeling of M_n

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 2, n \neq 3$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$
$n = 3$	4	5

Table 3.

Q.E.D.

A Young tableau is a sub graph of $P_m \times P_n$ obtained by retaining the first two rows of $P_m \times P_n$ and deleting the vertices from the right hand end of other rows in such a way that the lengths of the successive rows form a non increasing sequence.

Theorem 2.18. Let G be a graph obtained from a Young tableau which is obtained from the grid $P_n \times P_n$ (n odd), by adding an extra vertex above the top row of a Young tableau and joining every vertex of the top row to the extra vertex. Then G is difference cordial.

Proof. Consider the right corner vertex of the top row. Label that vertex by 1. Then assign the labels $2, 3 \ldots n$ to the preceding vertices of the top row. That is, the left corner vertex of the top row receive the label n. Then we move to the second row. Assign the label $n+1$ to the left corner vertex of the second row. Then assign the labels $n+1, n+2, \ldots 2n$ to the successive vertices of the second row. Then we move to the right corner vertex of the third row and label it by $2n+1$ and the preceding vertices of third row are labeled by $2n+2, 2n+3, \dots 3n-1$. Then we move to the left corner vertex of the fourth row and so on. Finally, assign the label p to the extra vertex. Obviously, this vertex labeling is difference cordial labeling. Q.E.D.

A difference cordial labeling of G with $n = 7$ is given in figure 1.

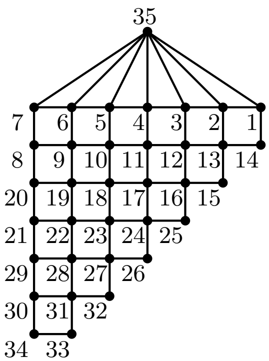

FIGURE 1.

A Mongolian tent $MT_{m,n}$ is a graph obtained from $P_m \times P_n$, n odd, by adding one extra vertex above the grid and joining every other vertex of the top row of $P_m \times P_n$ to the new vertex.

Theorem 2.19. The Mongolian tent $MT_{m,n}$ (n odd) is difference cordial.

Proof. Let $V(MT_{m,n}) = \{u, u_{i,j} : 1 \le i \le m,\ 1 \le j \le n\}$ and $E(MT_{m,n}) = \{uu_{i,j} : 1 \le i \le m,\ 1 \le j \le n-1\} \cup \{u_{i,j}u_{i,j+1} : 1 \le i \le m,\ 1 \le j \le n-1\} \cup \{u_{i,j}u_{i+1,j} : 1 \le i \le m-1,\ 1 \le j \le n\}$. The order and size of $MT_{m,n}$ are $mn+1$ and $2mn-m$ respectively. Define an injective map f from the vertices of $MT_{m,n}$ to the set $\{1, 2 \dots, mn+1\}$ as follows:

$$
\begin{array}{lcll}
f(u_{4i-3,1}) & = & 4n(i-1)+1 & 1 \le i \le \frac{m}{4} \quad \text{if} \quad m \equiv 0 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+3}{4} \quad \text{if} \quad m \equiv 1 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+2}{4} \quad \text{if} \quad m \equiv 2 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+1}{4} \quad \text{if} \quad m \equiv 3 (\text{mod } 4). \\
f(u_{4i-2,1}) & = & (4i-2)n & 1 \le i \le \frac{m}{4} \quad \text{if} \quad m \equiv 0 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-1}{4} \quad \text{if} \quad m \equiv 1 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+2}{4} \quad \text{if} \quad m \equiv 2 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+1}{4} \quad \text{if} \quad m \equiv 3 (\text{mod } 4). \\
f(u_{4i-1,1}) & = & (4i-1)n & 1 \le i \le \frac{m}{4} \quad \text{if} \quad m \equiv 0 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-1}{4} \quad \text{if} \quad m \equiv 1 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-2}{4} \quad \text{if} \quad m \equiv 2 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m+1}{4} \quad \text{if} \quad m \equiv 3 (\text{mod } 4). \\
f(u_{4i,1}) & = & n(4i-1)+1 & 1 \le i \le \frac{m}{4} \quad \text{if} \quad m \equiv 0 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-1}{4} \quad \text{if} \quad m \equiv 1 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-2}{4} \quad \text{if} \quad m \equiv 2 (\text{mod } 4) \\
 & & & 1 \le i \le \frac{m-3}{4} \quad \text{if} \quad m \equiv 3 (\text{mod } 4).
\end{array}
$$

$$
\begin{array}{lclll}
f(u_{4i-3,j}) & = & f(u_{4i-3,j-1})+1 & 2\le j\le n & \\
 & & & 1\le i\le \frac{m}{4} & \text{if} \quad m\equiv 0(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+3}{4} & \text{if} \quad m\equiv 1(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+2}{4} & \text{if} \quad m\equiv 2(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+1}{4} & \text{if} \quad m\equiv 3(\text{mod } 4).\\
f(u_{4i-2,j}) & = & f(u_{4i-2,j-1})-1 & 2\le j\le n & \\
 & & & 1\le i\le \frac{m}{4} & \text{if} \quad m\equiv 0(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-1}{4} & \text{if} \quad m\equiv 1(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+2}{4} & \text{if} \quad m\equiv 2(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+1}{4} & \text{if} \quad m\equiv 3(\text{mod } 4).\\
f(u_{4i-1,j}) & = & f(u_{4i-1,j-1})-1 & 2\le j\le n & \\
 & & & 1\le i\le \frac{m}{4} & \text{if} \quad m\equiv 0(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-1}{4} & \text{if} \quad m\equiv 1(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-2}{4} & \text{if} \quad m\equiv 2(\text{mod } 4)\\
 & & & 1\le i\le \frac{m+1}{4} & \text{if} \quad m\equiv 3(\text{mod } 4).\\
f(u_{4i,j}) & = & f(u_{4i,j-1})+1 & 2\le j\le n & \\
 & & & 1\le i\le \frac{m}{4} & \text{if} \quad m\equiv 0(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-1}{4} & \text{if} \quad m\equiv 1(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-2}{4} & \text{if} \quad m\equiv 2(\text{mod } 4)\\
 & & & 1\le i\le \frac{m-3}{4} & \text{if} \quad m\equiv 3(\text{mod } 4).
\end{array}
$$

and $f(u) = mn + 1$. The following table 4 shows that f is a difference cordial labeling of the Mongolian tent $MT_{m,n}$.

Nature of m	$e_f(0)$	$e_f(1)$
$m \equiv 0\,(\text{mod } 2)$	$\frac{2mn-m}{2}$	$\frac{2mn-m}{2}$
$m \equiv 1\,(\text{mod } 2)$	$\frac{2mn-m+1}{2}$	$\frac{2mn-m-1}{2}$

TABLE 4.

Q.E.D.

Finally, we investigate the n-cube.

Theorem 2.20. $K_2 \times K_2 \times \cdots \times K_2$ (n times) is difference cordial.

Proof. Let $G = K_2 \times K_2 \times \ldots \times K_2$ (n times). Let $V(G) = \{u_i, v_i, w_i, x_i : 1 \le i \le n-1\}$ and $E(G) = \{u_iv_i, v_ix_i, x_iw_i, w_iu_i : 1 \le i \le n-1\} \cup \{u_iu_{i+1}, v_iv_{i+1}, w_iw_{i+1}, x_ix_{i+1} : 1 \le i \le n-2\}$. Define a map $f : V(G) \to \{1, 2, \ldots, 4n-4\}$ by $f(u_i) = i$, $1 \le i \le n-1$, $f(v_{n-i+1}) = n-1+i$, $1 \le i \le n-1$, $f(w_i) = 2n-2+i$, $1 \le i \le n-1$, $f(x_{n-i+1}) = 3n-3+i$, $1 \le i \le n-1$. Since $e_f(0) = e_f(1) = 4n-6$, f is a difference cordial labeling of G. Q.E.D.

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